IFS ver5 Tutorial

Introduction:

It is easy to represent the geometric operations of translation, reflection, rotation and contraction as algebraic statements. These statements, which are transformations from the xy plane onto the xy plane, can be applied randomly to an arbitrarily selected point to produce a sequence of points known as an orbit.

The whole process is a bit like playing golf. You start with a point, this is your ball, and some transformations, these are your clubs. From your initial position you apply one of the transformations and reflect, translate, scale or whatever. This is your shot. This transformation, however, is chosen at random from your set of transformations. From this new position you choose a transformation at random and take another shot. This operation is repeated for as many iterations as you like. As you take the shots they appear to follow a path with no rhyme or reason. However, if you look back at a plot of the first few thousand shots then very often you will see an interesting fractal or geometric image.

These images are admittedly not as gaudy as their multicolored cousins drawn with a program like fractal explorer. But they can be quite intricate and interesting. They are a lot of fun to create because rather than prospecting for "pretty" pictures you have to use basic principles of geometry to construct the functions which will draw your pictures. Moreover the range of images from spirals and geometric designs to ferns and trees is limitless.

IFS ver5 is free and available for use or download from the downloads page of this site. It is a modification of software that Michael Frame uses in his fractal geometry courses at Yale University. You can get to the Yale fractal site from the links page of this site.

How Random IFS Works:

All images begin with a unit square. Figure 1. shows a screen capture of IFS ver5 with three transformations. Since there are 0 iterations, the image shown is that of the unit square. Figure 2. is the result of 1 iteration of each of the transformations applied to the unit square. The transformations are easy to explain:

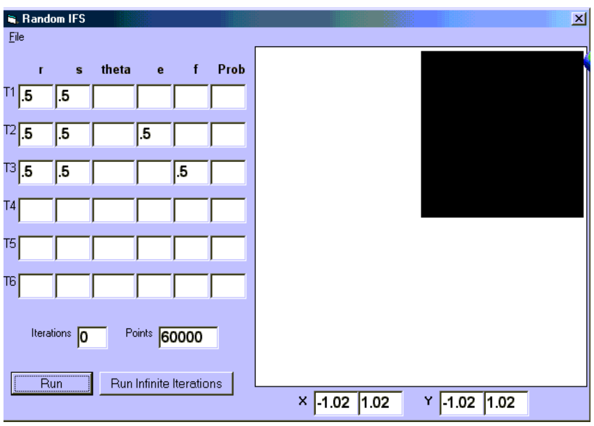

Figure 1.

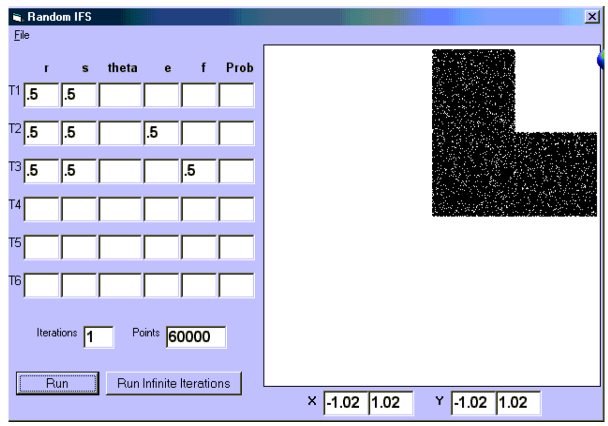

Figure 2.

The values for r and s are scalings in the x and y directions respectively.

The values for s and t are translations in the x and y directions respectively.

The value for theta rotates the image counterclockwise around the origin.

Translations are done after scalings and rotations.

Negative values for r and s will reflect in the x and y axes respectively.

A negative value for theta will produce a clockwise rotation.

The transformations from our example have easy algebraic formulations–

$T_1 (x, y) = (.5x, .5y)$
$T_2 (x, y) = (.5x + .5, .5y)$
$T_3 (x, y) = (.5x, .5y + .5)$

All three of these transformations contract the unit square by half in the x and y directions. The second also moves this shrunken square .5 units to the right while the third moves it .5 units up. So the image of the unit square under each of these transformations produces one of the three squares making up the image in figure 2. after one iteration.

The images of each of these three squares under each of the transformations will produce 9 squares. The images of those 9 under the transformations will produce 27 squares, etc.

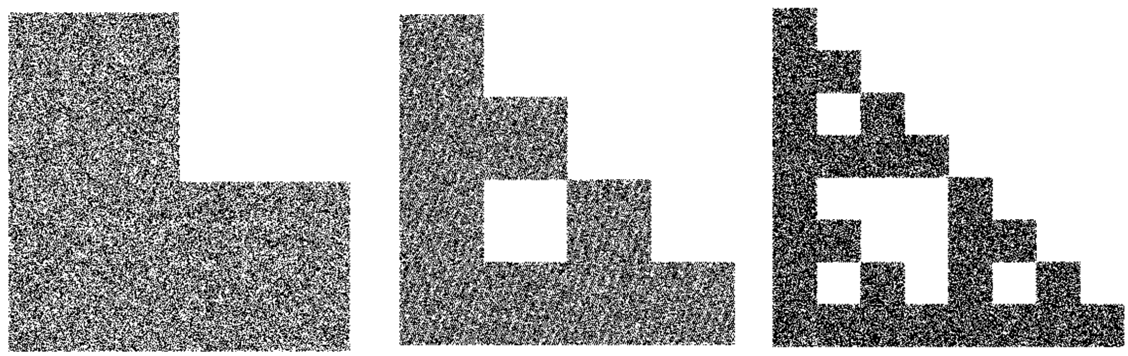

Figure 3.

Continuing in this fashion we produce a sequence of sets whose intersection is the familiar Sierpinski gasket.

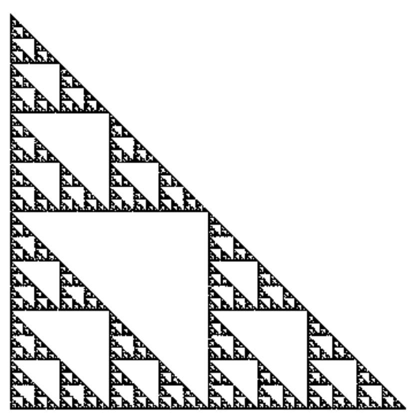

Figure 4.

Now, if we begin an orbit with some point (x_0, y_0) in the unit square then the image of any one of the three transformations will give us a point (x_1, y_1) in one of the three smaller square making up the image after one iteration. Applying a transformation to (x_1, y_1) will give us a point (x_2, y_2) in one of the nine squares making up the image after two iterations, and so forth. Each new point (x_{n+1}, y_{n+1}) lies in a smaller square than its predecessor (x_n, y_n). This sequence, which we call an orbit, does not converge to a point but it does get arbitrarily close to the Sierpinski gasket.

This is how all pictures with IFS ver5 are drawn. We create a transformation T by entering values for r, s, 2, e, f and prob. The actual transformation can be written as

$$(x_{n+1}, y_{n+1}) = T(x_n, y_n) = (r \cos 2\ x_n - s \sin 2\ y_n + e , r \sin 2\ x_n + s \cos 2\ y_n + f)$$

with *r* # 1 and *s* # 1. That is, r and s are contraction and reflection factors for x and y, 2 is the angle of rotation about the origin and e and f are translations parallel to the x and y axes. If no values are entered in the probability columns then all transformations will be performed with equal likelihood. Otherwise the transformations are performed according to their probability value relative to the sum of all of the probability values. There are two options, **Run** and **Run Infinite Iterations**. **Run** applies the transformations first to the unit square and shows the image after any desired number of iterations. **Run Infinite Iterations** shows up to 999,999 points of the orbit of a randomly selected initial point from the unit square.

<u>Some Tips For Drawing Pictures</u>:

It doesn't take many transformations to make an interesting picture. While we are constructing the orbit of a point it helps to remember what each transformation is doing to a shape in the xy plane. For example in figure 5. we see that we are either contracting and translating or we are rotating. We are never doing both at the same time. However, both operations seem to be getting done at the same time.

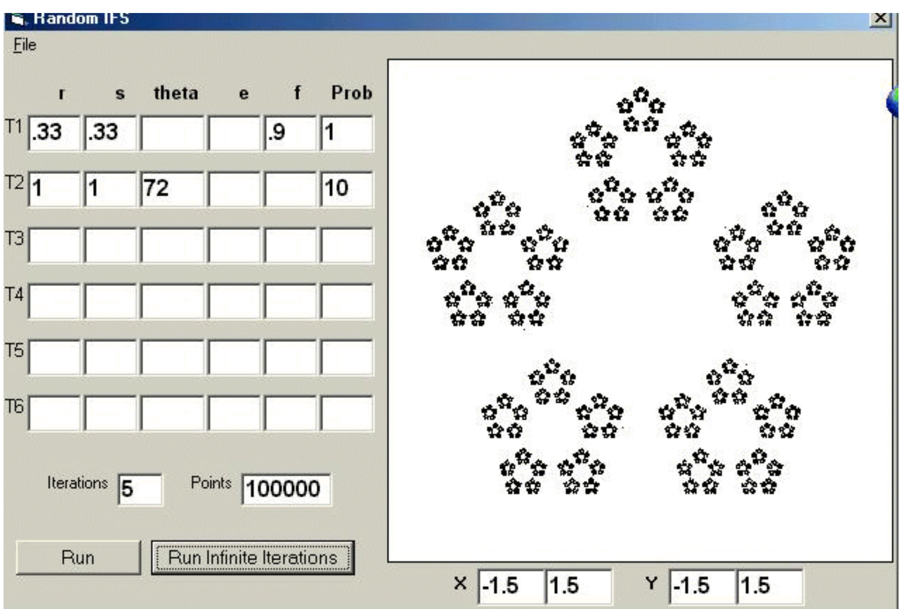

Figure 5.

To get a sense of why we get the pentagonal figure in figure 5. imagine that we have contracted and translated the unit square. If we then rotated it four times we would get our five squares positioned symmetrically about the origin. This arrangement of squares, however, can be shrunken and translated giving us our arrangement of five squares in the position occupied by the first square. If we now rotate four times, we will get our small arrangement of five squares positioned symmetrically about the origin. Continuing with this process we end up with our arrangement of five pentagons with each of the pentagons an arrangement of five pentagons etc. As we mentioned above, both transformations are never performed at the same time but both seem to get done at the same time.

Note also that the probability values for the two transformations are different. In order to get all 5 pentagons filled in nicely it is necessary to have the rotation transformation performed more often that the shrink and translate one.

If we shrink a little more and rotate a smaller amount then we get the 10 ringed image.

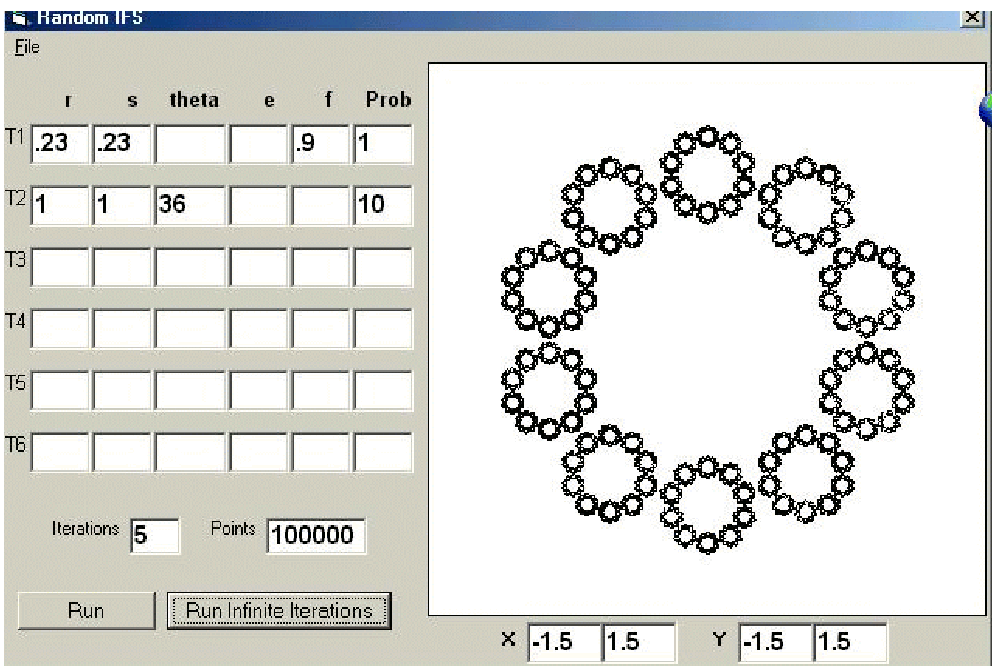

Figure 6.

We can change the shape of the rings by using different contraction ratios for x and y. Letting r = .3 and s = .01 we can make a "toothpick"and then move it and rotate it to make a decagon.

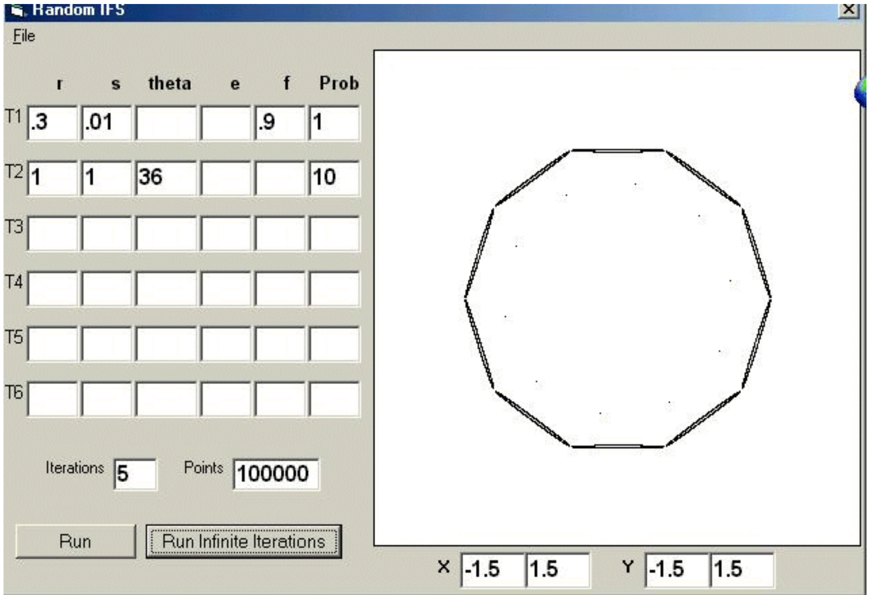

Figure 7.

Turning the toothpicks and making them a little longer isn't too interesting.

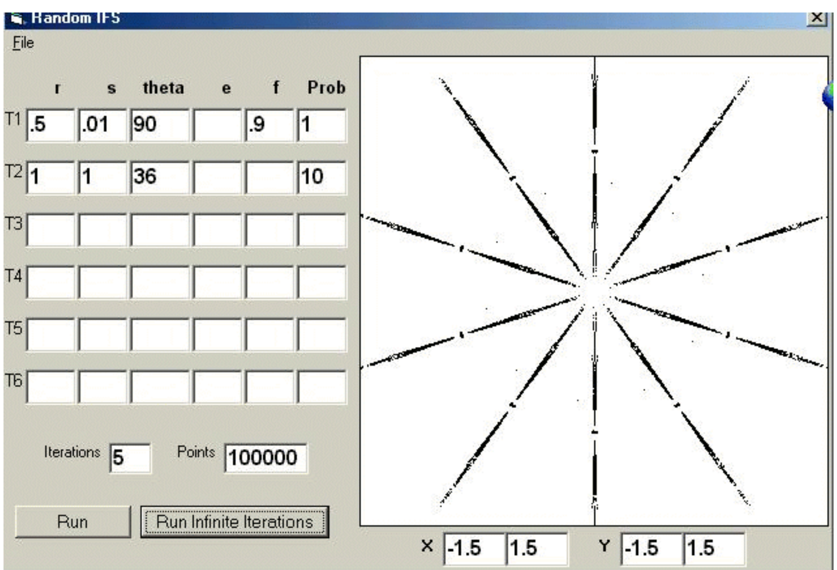

Figure 8.

But if we fatten up the toothpicks it gets more interesting.

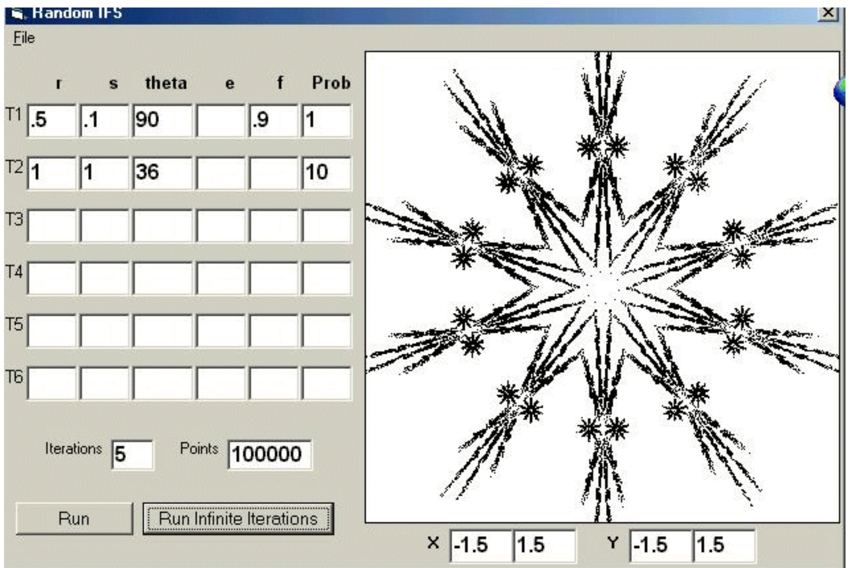

Figure 9.

Then if we turn the toothpicks back we end up with overlapping flat rings

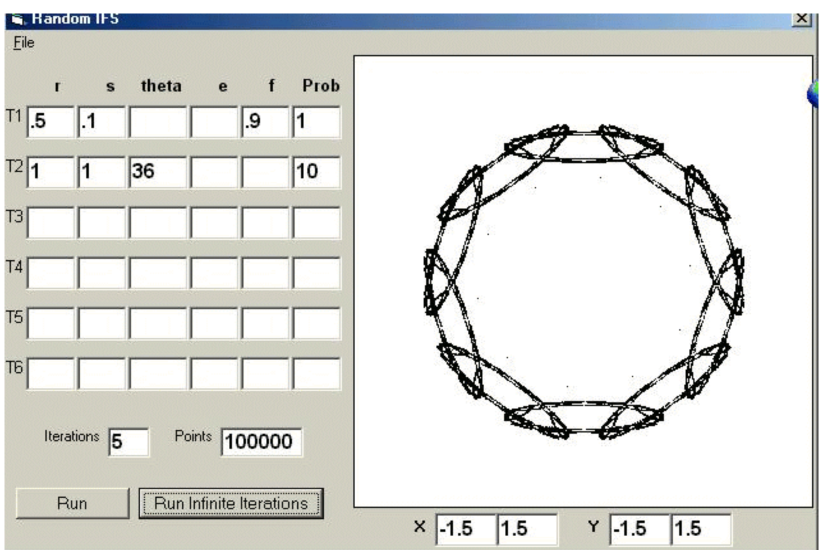

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Figure 10.

Shapes can be created and rotated. Here a big toothpick is rotated to form a box. Then with a third transformation a smaller copy of the box is placed in the center and rotated.

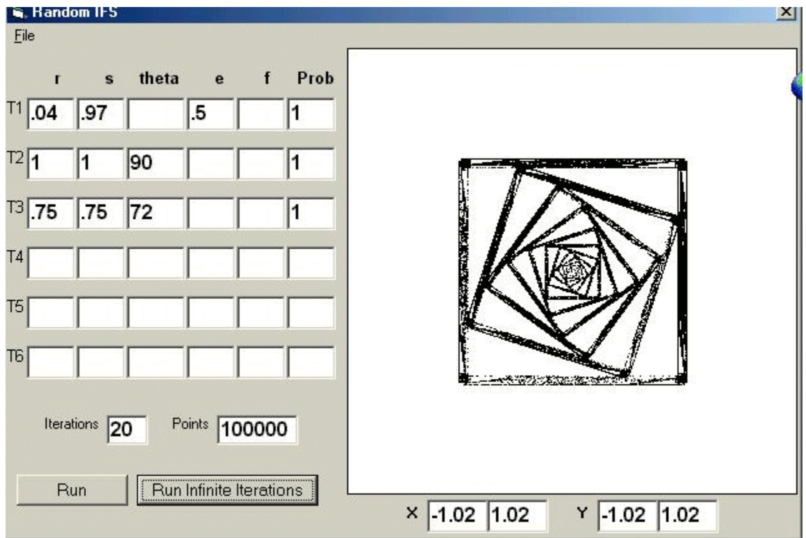

Figure 11.

Toothpicks can be made to spiral.

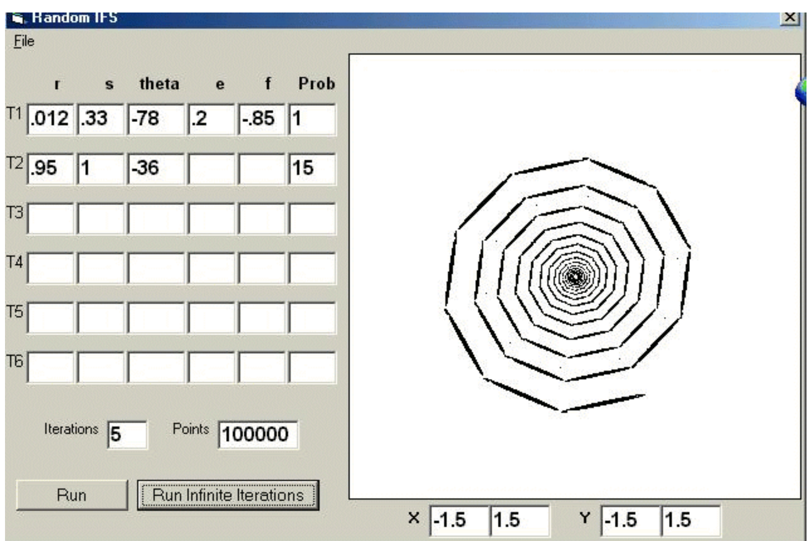

Figure 12.

Or lines can be rotated to make stars.

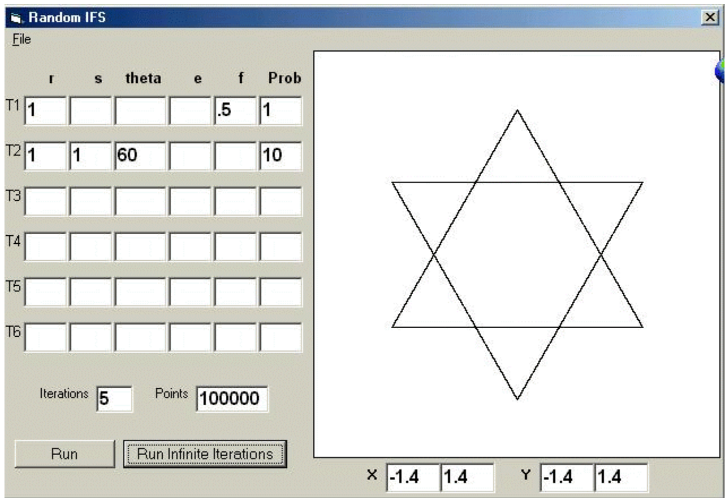

Figure 13.

You get interesting effects if you rotate and contract at the same time.

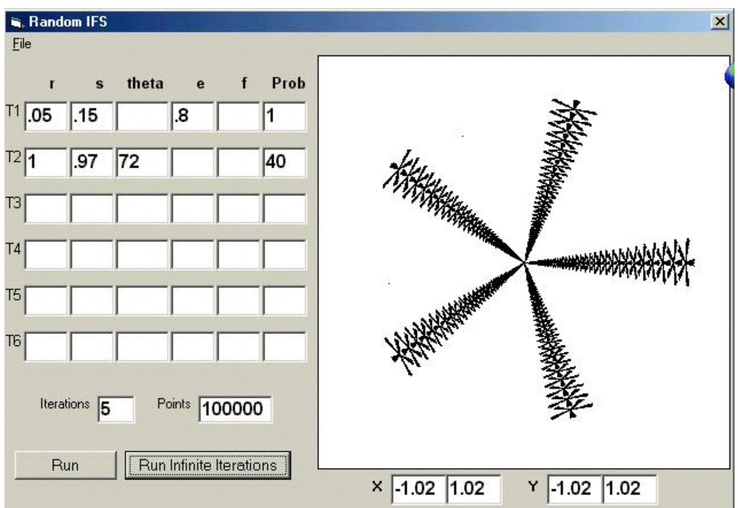

Figure 14.

However, if the angle of rotation is not a divisor of 360 then your image can spiral.

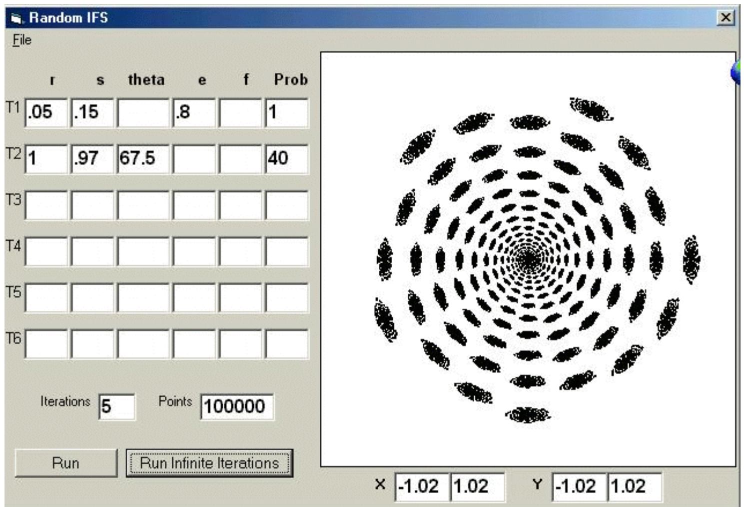

Figure 15.

If we contract a little more and enlarge the thing that we are rotating then we get a prettier spiral.

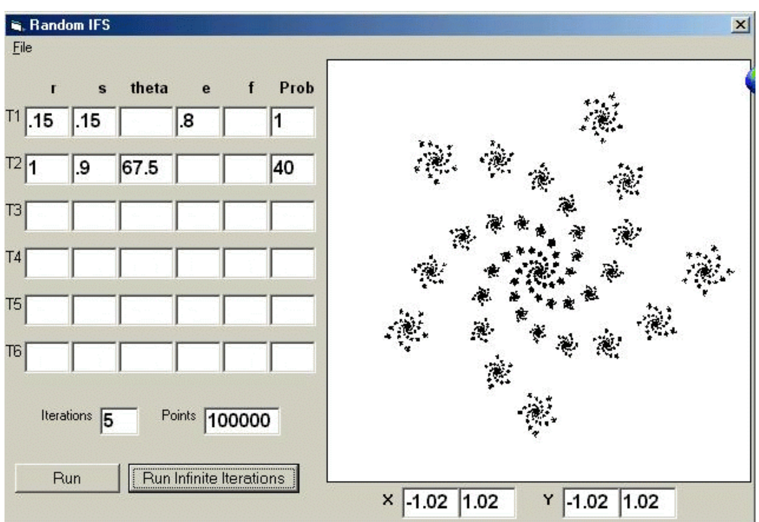

Figure 16.

With a couple of different transformations we can get a sea urchin.

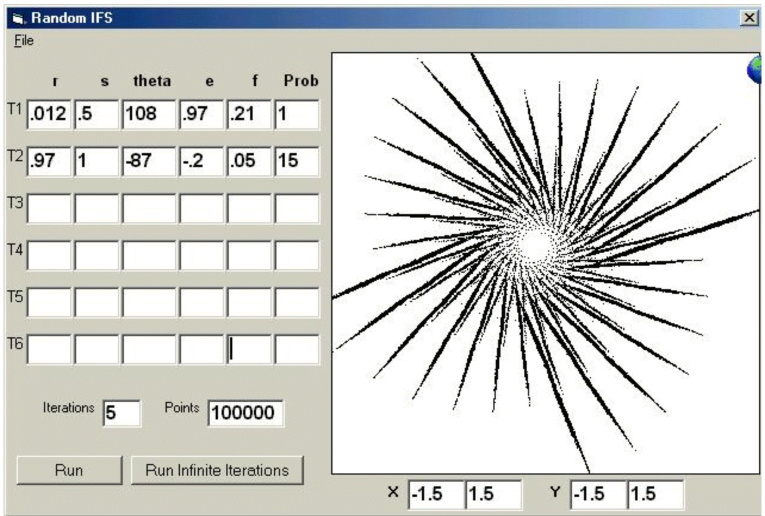

Figure 17.

This small sample of transformations with their images is barely a glimpse of the possible images that can be obtained. The way to learn to make images is to practice and try things out. You will occasionally have a great image "turn to sand" as you make modifications. But that is all part of the game. There are hundreds of IFS images in the various galleries on this site to show you the possibilities. New and improved versions of IFS will have sample images that you can select and modify in a "live" IFS session. As you gain experience with IFS you may find it a compelling and fun diversion. You may even exclaim as one of my fractal geometry students did that "This is better than solitaire."